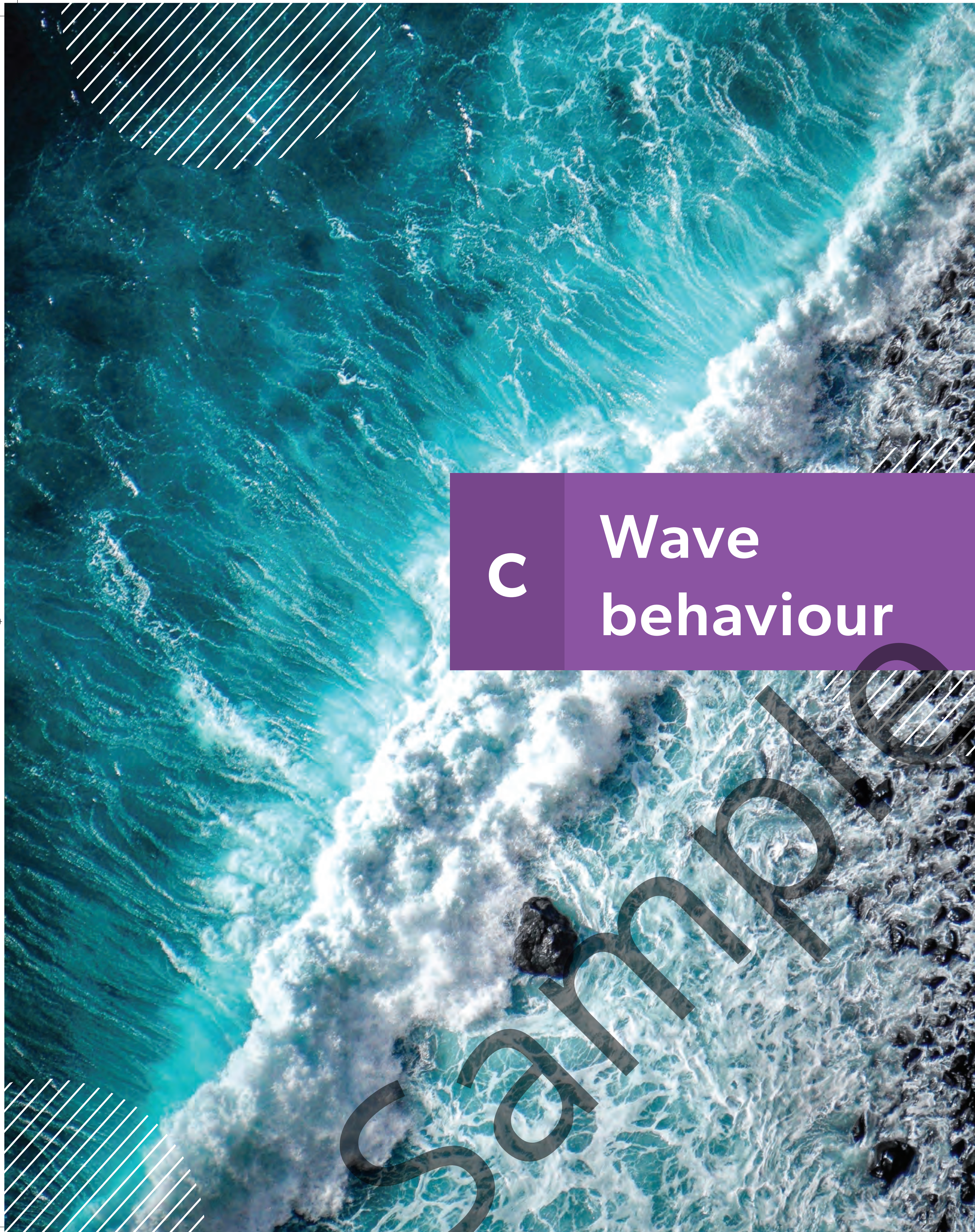


C Wave behaviour

Introduction

Every year, a human heart beats more than 50 million times – an oscillation that leads to the movement of blood around our bodies. Motions such as this and the beating of a hummingbird's wings are periodic. The pattern of the motion repeats again and again, sometimes with the same fixed time interval between each repeat. This Theme deals with the physics of such a motion, known as an oscillation.

We begin in Topic C.1 with a detailed analysis of one important type of oscillation: simple harmonic motion (SHM). SHM has a fundamental importance. Complex oscillations can be described as the combined sum of many simple harmonic motions. This summation is important in many fields of science and engineering.

Oscillations lead to the production and transmission of mechanical waves. Waves come in many forms: Sound waves transmit through all materials and enable us to hear. Earthquake waves travel through the Earth. Our knowledge of wave theory enables us to understand and predict the behaviour of many man-made and natural phenomena.

Topic C.2 begins the work on waves themselves with the description of a model for wave motion. Topic C.3 looks at the effects that occur when waves interact with different media and with each other. Topic C.4 continues with a description of the mechanisms that lead to standing waves – an important part of the production of sound in musical instruments. The Theme ends with Topic C.5 that deals with the Doppler effect when waves are emitted and detected by sources and observers moving relative to each other.

The concepts of **particles** and **energy** are inextricably linked throughout Theme C. Waves transfer energy but the medium that carries the wave is undisturbed when the wave has gone through.

A mechanical wave is made up of the movement of particles. The particles are the medium for the wave. However, electromagnetic waves do not have a particulate nature and do not require a medium. The physics of this wave transfer is significantly different from that of mechanical wave motion. These differences have led to profound changes in our understanding of spacetime.

Two features of physics that have underpinned the theory of oscillations are **observations** and **measurements**. Galileo is said to have used his own pulse to time the slow swings of the huge candelabra in the cathedral of Pisa. He recognised that, whatever the amplitude of the swing, the period was constant. To what extent would we regard these as reliable observations today?

C.1 Simple harmonic motion

What makes the harmonic oscillator model applicable to a wide range of physical phenomena?

Why must the defining equation of simple harmonic motion take the form it does?

How can the energy and motion of an oscillation be analysed both graphically and algebraically?

Simple harmonic motion is an oscillation with an unchanging amplitude and frequency and which never ends. No energy is transfering from the oscillating system. It may seem strange to learn about such a specific type of motion, but there is a good reason. Joseph Fourier showed that any periodic motion could be regarded mathematically as a sum of individual simple harmonic motions. Study simple harmonic motion and you have studied more complex oscillations too. However, many oscillations are either purely or approximately simple harmonic. A buoy floating in the sea, a mass oscillating on a spring and a pendulum are just three common examples of this motion.

The motion itself is characterized by a simple defining equation. The acceleration of a system is directly proportional to the displacement of the system and acts opposite to the displacement direction. The equation contains only three quantities, including a constant of proportionality, but the way in which these interact generates oscillations. The constant of proportionality tells us about the time taken to complete one oscillation. The statement about direction is crucial too. It says that the further the object is from an equilibrium position, then the larger is the acceleration back towards the equilibrium point. This already suggests an oscillation of some kind.

The oscillation trades displacement for velocity, and potential energy for kinetic energy. When the system is far from equilibrium it is travelling slowly. Around the equilibrium point it is moving quickly so that its momentum carries it through equilibrium to the other half of the cycle. At this point, the force on the system (and therefore the acceleration) reverses direction, once more acting towards the equilibrium point.

Our defining equation also leads to sets of equations linking the displacement, velocity and acceleration of the oscillating system with time. This means that we can go on to use knowledge from Theme A to describe the energy transfers in the oscillating system too. These can also be expressed in terms of time and distance.

Finally, graphical representations of energy–time and displacement–time can be linked to real examples of simple harmonic motion. This allows us to confirm that our equation for harmonic motion and the predictions it makes are a good fit to the real oscillations that we observe in a practical context.

In this topic, you will learn about:

- oscillations and simple harmonic motion
- the defining equation of simple harmonic motion
- the conditions for simple harmonic motion
- displacement, amplitude, time period, frequency, angular frequency and equilibrium position
- phase angle (AHL)
- the mass–spring system and the simple pendulum
- energy changes during an oscillation
- kinematic and energy calculations involving simple harmonic motion
- kinematic and energy calculations involving simple harmonic motion. (AHL)

Introduction

In this topic, you will meet the language of oscillation and consider the harmonic oscillator, usually referred to as simple harmonic motion. True simple harmonic motion can only be obtained in some systems under certain limited conditions, such as small displacements. Nevertheless, you can still use simple harmonic motion as a model in these systems, if you accept the conditions and the limitations they impose.

▲ Figure 1 Knowledge of simple harmonic motion led to the development of the pendulum clock. For about 300 years, pendulum clocks were the most precise clocks available. This is a sidereal clock used to help make astronomical observations.

ATL Drafting, revising and improving academic work

Joseph Fourier was a French mathematician and physicist who lived from 1768 to 1830. In 1807, he read a paper to the Paris Institute "*On the Propagation of Heat in Solid Bodies*". In it he used a mathematical method to reduce a complicated oscillation to a series of sine waves.

You can try this for yourself. Use a graphical calculator or a spreadsheet to help plot the function $y = \sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \cdots$. You can add further terms of $\frac{1}{n}\sin nx$ for odd values of the integer n. It does not require very many terms in the series to show that the series approaches a square-wave. You could also try the even terms to see what happens.

However, Fourier's paper did not convince everyone in the audience. He had relied on intuition in places and there were some gaps in his logic. His mathematical method also contradicted some of the work of one of the examiners in the audience—Joseph-Louis Lagrange.

To settle the matter, a prize problem was set in 1810 and Fourier submitted his original paper along with some new work. There was only one other paper, and Fourier won the competition. But the feedback (possibly from Lagrange) was not entirely favourable, and the result was that Fourier's work was not published until 1822.

Fourier's method of splitting a signal into sinusoidal waves of different frequencies is widely used today and is the principle behind the spectral analysis of sound.

Oscillations

Many oscillations in science and engineering are **isochronous**. This means that the oscillation repeats, taking the same repetition time irrespective of its size. This is important because, unless energy is transferred to them, real oscillating systems "run down" and eventually stop. The **amplitude**—the maximum displacement—of the system decreases when it transfers energy to the environment.

▲ Figure 2 A swinging pocket watch is an example of a simple pendulum oscillating with approximate simple harmonic motion.

Technology for timing

Galileo is reputed to have first observed that the time period of a simple pendulum did not depend on its amplitude (provided that the amplitude remained small). The story is that when he was about 17 years old, he was bored during a service in Pisa Cathedral and observed the way that the chandelier swung as the wind blew it. He compared the time for the swings with his pulse. Sometimes the wind blew the chandelier into large oscillations and sometimes the oscillations were small. However, the number of oscillations in a certain number of pulse beats was always the same.

While it is likely that other scientists may have observed that a pendulum's period does not change with amplitude, Galileo was perhaps one of the first to use the pendulum in experiments. As a result, scientists could now measure time and hence other quantities such as speed. Without this timing mechanism, experiments in mechanics would have been impossible. The importance of experimental evidence in scientific knowledge was still a relatively new concept at this time, and the increased ability to conduct experiments increased the importance of this evidence.

How else has technology affected the value we place on different forms of knowledge?

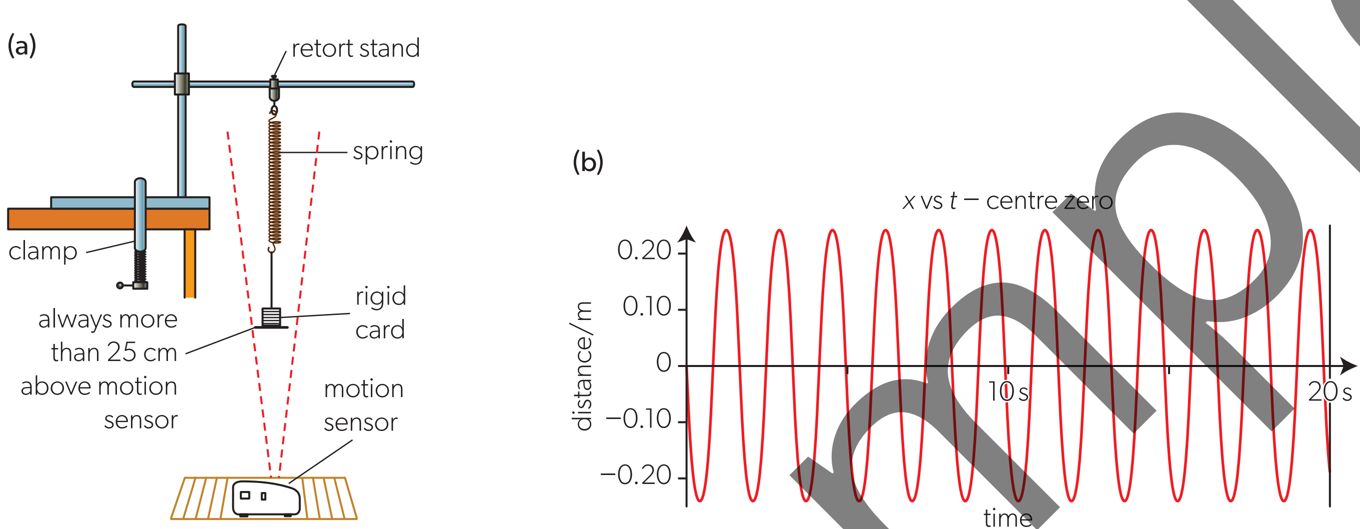
Figure 2 shows a pocket watch oscillating about its centre (equilibrium) position from the maximum position on one side to the other. The watch is illuminated with a flash that occurs every 0.25 s and so it takes 1.0 s for the watch to complete each oscillation (to go from one side to the other and back again). A simple pendulum only performs approximate simple harmonic motion which changes at large amplitudes of swing. Nevertheless, a timepiece can be governed to make it into an isochronous oscillator.

Defining periodic motion

Before we can develop the mathematics of simple harmonic motion, we need a technical language.

To illustrate the terms we use, imagine an experiment with a mass hanging at the end of a spring (Figure 3(a)). The position of a small card attached to the mass is detected by a motion sensor on a data logger that produces a graph of displacement against time for the mass (Figure 3(b)).

- The mass with its card is shown on the left in its **equilibrium position**. This is the position it adopts when at rest.
- The mass–spring system oscillates when displaced vertically and released (it takes both a spring and a mass to oscillate; hence the word "system").



▲ Figure 3 (a) The experimental arrangement and (b) the resulting displacement–time graph for an illustration of simple harmonic motion.

The position of the oscillator at any moment in time is known as the **displacement** x. As in Theme A, displacement is a vector that can be positive (when the mass is above the equilibrium position here) or negative (when it is below). Once the positive direction has been chosen as upwards, then it must be used in a consistent way for all vector quantities in the oscillation, including forces, velocities, displacements and accelerations.

- The maximum displacement of the oscillator is known as the **amplitude** x_0. This amplitude is measured from the equilibrium position to the extreme (largest) displacement. It is *not* the distance from one extreme to the other. Amplitude does not have a sign and is not a vector quantity.
- One complete **cycle** of the oscillation occurs when the mass (in this situation) goes from one position in the motion through the extreme position on the opposite side, back to the other extreme, finally moving through the original position *in the original direction*. It is easiest to understand this for the mass–spring system by starting at the equilibrium position. The mass goes down to the bottom, back through the equilibrium, moving upwards, and to the top. Then it goes down through the equilibrium again. The cycle only ends with this second transit through the equilibrium. Trace this motion out on the graph (Figure 3(b)). There are six cycles in 10 s.
- The time taken to complete one cycle is known as the **time period**, T. For the isochronous mass–spring system, the time period (often shortened to **period**) does not depend on where the cycle starts or on the amplitude.
- The **frequency** f of the oscillation is the number of cycles that the system goes through in one second. Thus

$$f = \frac{1}{T}$$

The unit of frequency is the hertz (Hz), which is the same as s^{-1}.

Practice questions

1. Which of the following quantities describing an oscillation can be negative?
   A. displacement B. amplitude C. period D. frequency
2. A mosquito flaps its wings at a frequency of 580 Hz. Calculate the period of mosquito's flaps.
3. An object undergoes simple harmonic motion with a period of 0.40 s. The distance between the extreme positions of the object is 6.0 cm. Calculate:
   a. the frequency
   b. the amplitude.

Applying the definitions

These definitions apply to many repetitive phenomena such as the rhythm of a human heart. Figure 4 shows the electrocardiograph of a healthy heart that is beating at 65 beats per minute, a frequency of $\frac{65}{60} = 1.08$ Hz. This means that T for the graph is $\frac{1}{f} = \frac{1}{1.08} = 0.92$ s. The overall height of the voltage spike from 0 V, shown as A in Figure 4, is the amplitude signal output by this sensor.

Worked example 1

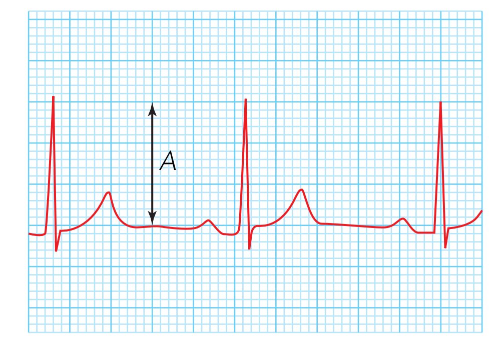
The pendulum of a wall clock completes 25 oscillations in 30 s. Calculate:

a. the period
b. the frequency of the oscillations.

Solutions

a. $T = \frac{30}{25} = 1.2$ s

b. $f = \frac{1}{T} = \frac{1}{1.2} = 0.83$ Hz



▲ Figure 4 The normal heart rhythm of an adult male. The graph shows the pd measured using a voltage sensor attached to the chest wall.

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Investigating a mass — spring system

- Tool 2: Use sensors.
- Tool 2: Represent data in a graphical form.
- Inquiry 1: Develop investigations that involve hands-on laboratory experiments, databases, simulations and modelling.
- Inquiry 1: Design and explain a valid methodology.

In this investigation, an ultrasound motion sensor is used to monitor the position of a mass suspended from the end of a long spring. The data logger software processes the data to produce a graph showing the variation of displacement with time.

- Arrange the apparatus as shown in Figure 3(a). The system needs to have a period of at least 1.0 s. Avoid reflections from the surroundings by keeping objects well away from the apparatus.
- Put the mass into oscillation by displacing it vertically and then releasing it.
- Set up the data logger so that it is triggered to start reading at a given displacement value.
- Use software to plot graphs of velocity and acceleration (in addition to displacement) against time.
- Devise an investigation to find out how the time period of the oscillation varies with:
  - spring constant *k*
  - mass *m* on the spring.
- You may wish to carry out a preliminary set of runs to get an idea of the relationships between *k* and *T*, and between *m* and *T*. Try doubling the mass or quadrupling it to see the effect on *T*. Two or more identical springs can be joined together in series or in parallel to vary *k*. (Hint: look at page 59 to remind yourself how *k* depends on the arrangement of springs).
- You can also perform similar investigations with other oscillations, such as a mass swinging from side to side at the end of a long string — a simple pendulum.

Global impact of science

Heinrich Hertz, for whom the frequency unit was named, was a German physicist working in the mid-19th century. He demonstrated the existence of electromagnetic radiation in the radio wavelengths and (famously) suggested that his work had no future application! Within 60 years, the Italian nobleman Count Marconi had sent messages across the Atlantic Ocean using radio waves. Hertz, unfortunately, never lived to see the application of radio waves, as he died in 1894 aged 36.

There is a direct link between the frequency of simple harmonic motion and the frequencies of the electromagnetic radiation that Hertz identified. His waves consisted of oscillating electric and magnetic fields that are modelled as sinusoidal variations just like those of an oscillating spring.

Simple harmonic motion

The variation with time of the displacement of the mass–spring system shown in Figure 3(b) is regular and simple. This is a negative sine curve (making the mass go upwards first will make this a positive sine curve). Oscillations that follow this model with a sinusoidal displacement–time graph are undergoing **simple harmonic motion**.

There are two requirements for motion to be simple harmonic. Both relate to the restoring force (and therefore the acceleration) acting on the system.

- The size (magnitude) of the force (acceleration) must be proportional to the displacement of the object from a fixed point.
- The direction of the force (acceleration) must be towards the fixed point.

Newton's second law of motion links acceleration and force in these statements.

At the equilibrium position, the weight of the mass is equal and opposite to the tension in the spring (assuming that the spring has negligible mass).

When the spring obeys Hooke's law (Topic A.2, page 57), then $F = -kx$, where F is the restoring force on the spring, k is the spring constant and x is the spring extension.

Substituting for F means that, for simple harmonic motion:

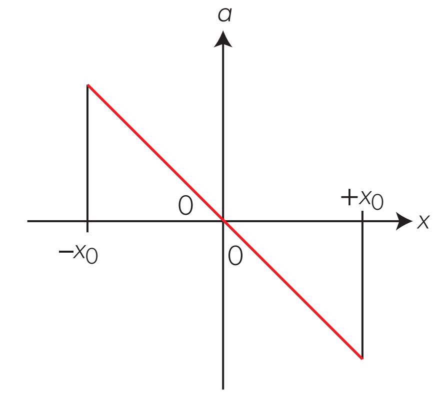
$$a = -(\text{constant})^2 \times x$$

You can find out more details of electromagnetic radiation in Topic C.2.

The constant is squared. This forces it to be positive, so that the minus sign always indicates that the displacement and acceleration vectors are in opposite directions. As a result, this equation now agrees with both of the requirements for simple harmonic motion.

In simple harmonic motion, the system is always accelerated towards the centre of the motion — the equilibrium position. When the mass is moving away from the equilibrium position, the system is slowing the mass down, accelerating it towards the equilibrium position. When the mass has reached the extreme of the motion, the system still accelerates it towards the equilibrium position, but now the speed of the motion increases until it reaches a maximum in the motion's centre.

This is summed up in Figure 5, which shows the variation of acceleration with displacement for any simple harmonic motion, not just the mass–spring system here. The gradient of the graph is negative as expected.



▲ Figure 5 A graph of the variation of acceleration with displacement for simple harmonic motion. The graph is a straight line of negative gradient going through the origin.

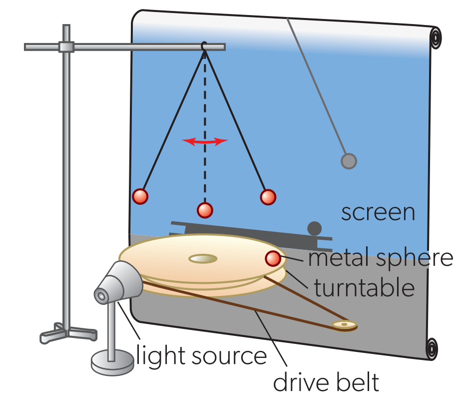
We need to know more about the constant in the defining equation. It is often written as

$$a = -\omega^2 \times x$$

with the constant as ω. This makes an important link between simple harmonic motion and the circular motion of Topic A.2.

Angular frequency

The oscillation of the pendulum can be compared with circular motion using the apparatus shown in Figure 6.



▲ Figure 6 The projection of a ball moving in a horizontal circle onto a vertical plane gives the same motion as a simple pendulum performing simple harmonic motion.

Two metal spheres are used, one acting as the mass for the pendulum. The other sphere is mounted on a horizontal turntable that rotates at a constant angular speed. The length of the string is adjusted so that the time period T of the simple harmonic motion oscillation is the time taken for the turntable to rotate once. When the arrangement is illuminated from the side, the two spheres move together and are synchronized on the screen. The circular motion is projected onto a vertical plane (the screen) and has the same pattern of movement as a pendulum when viewed in the same vertical plane.

The angular speed of the rotating sphere is

$$\frac{\text{angular displacement in radians}}{\text{time for one rotation}} = \frac{2\pi}{T}$$

In Topics A.2 and A.4, the quantity angular speed was given the symbol ω and therefore

$$\omega = \frac{2\pi}{T}$$

Putting this all together gives

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

The same is true for ω in the simple harmonic motion equation, but here the quantity is known as **angular frequency** because it has the unit s^{-1} equivalent to the hertz (Hz). As before, although this is rad s^{-1}, the radian is ignored because it is a unitless ratio.

Because ω is linked to T, which depends only on the properties of the harmonic oscillator, it also links the magnitude of the acceleration of the oscillator to its displacement. To show this link in more detail, we will look at two oscillators in detail: the mass–spring system and the simple pendulum.

How can circular motion be used to visualize simple harmonic motion?

The demonstration above shows the close link between circular motion and simple harmonic motion. One can be regarded as a one-dimensional projection of the other. The link extends beyond the purely practical, however, as the mathematics of circular motion from Topic A.2 and the mathematics of simple harmonic motion are themselves closely related. Similar physical quantities are defined in the same way in both.

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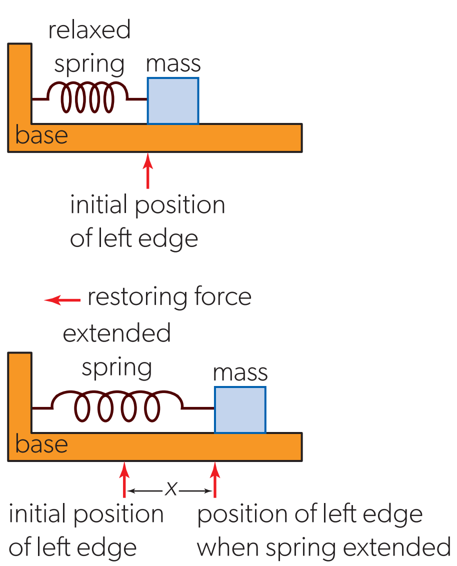
How can the understanding of simple harmonic motion apply to the wave model? (NOS)

Topics C.2 and C.3 deal with waves—periodic movements of interconnected individual particles that transfer energy. There must be an agent that generates the waves and this could easily be an object moving in a circle. The waves in deep oceans are linked to a circular motion of water that becomes an up-and-down motion of the surface. The mathematics developed in this Topic applies to the wave motions later.

▲ Figure 7 Waves on the surface of the ocean are caused by the circular motion of the water.



▲ Figure 8 A mass–spring system.

The mass–spring system

The mass–spring system here is a mass on a horizontal frictionless surface oscillating at the end of a spring. This is known as "exact simple harmonic motion" when the spring obeys Hooke's law. The horizontal case is easier to analyse than when the spring hangs vertically. (You can analyse the vertical case for yourself, remembering to include the weight of the mass as part of the net force that acts on the spring.)

The force F_H acting on the spring is directly proportional to its extension x: $F_H = -kx$ (from Topic A.2) and acts to return the spring to its equilibrium position. Therefore $ma = -kx$. When the positive direction is defined to be to the right and the mass is displaced to the right, the force must be directed to the left. The negative sign shows this.

This equation re-arranges to $a = -\left(\frac{k}{m}\right)x$ and shows the shape of the simple harmonic motion equation with its negative sign and positive constant inside the brackets.

Therefore, $\omega^2 = \frac{k}{m}$ and $\omega = \sqrt{\frac{k}{m}}$, leading to

$$T = 2\pi\sqrt{\frac{m}{k}}$$

as the equation for the time period of a mass–spring system.

The simple pendulum

A simple pendulum consists of an object on the end of a string of negligible mass that is swinging in a vertical plane. The pendulum obeys simple harmonic motion provided that the angle of swing from the vertical is small (<10°).

The string has a length l and is displaced with its bob of mass m through a vertical angle θ (Figure 9). When released, the bob moves with time period T.

The restoring force that pulls the bob back to the equilibrium position is $-mg\sin\theta$. The negative sign is because θ is measured to the right (anticlockwise on the diagram), but the restoring force is to the left (clockwise).

So $-mg\sin\theta = ma$, leading to $a = -g\sin\theta$.

The length of the arc from the equilibrium position to the bob is x, so

$$\theta = \frac{x}{l} \text{ and } a = -g\sin\left(\frac{x}{l}\right)$$

giving

$$a = -\frac{g}{l}x$$

providing that $\theta < 10°$.

You can check, using your calculator, that when $\theta < 12°$ (about 0.2 rad), then $\sin\theta$ and θ are within 1% of each other when calculated using radian measure.

Thus, $\omega^2 = \frac{g}{l}$ and $\omega = \sqrt{\frac{g}{l}}$, with

$$T = 2\pi\sqrt{\frac{l}{g}}$$

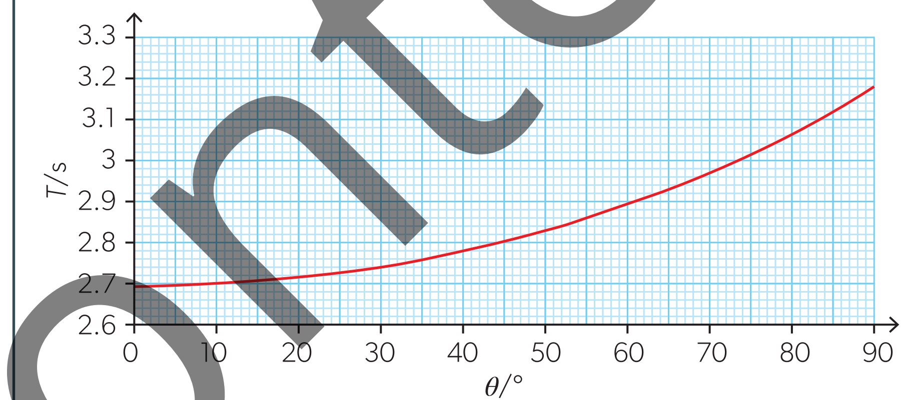
which is the equation for the time period of a simple pendulum.

Analyses such as this can be carried out for many more types of oscillator too, including floating cylinders bobbing up and down on the flat surface of a lake .

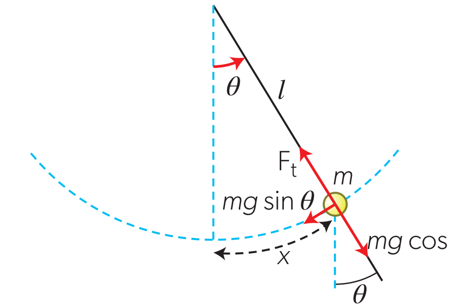
Data-based questions

As you have seen, a simple pendulum obeys simple harmonic motion (i.e. it is isochronous) provided that the amplitude is small (less than 10°). What happens if the pendulum swings through a larger amplitude?

The graph shows the variation of the time period T with angle θ for a pendulum of length 1.8 m.



- Use the graph to estimate the percentage difference in T when the pendulum swings with $\theta = 80°$ and when $\theta = 10°$.
- A student measures the time period of the pendulum by using oscillations with $\theta = 10°$. Explain why it would not be appropriate to give this measurement to 3 decimal places.
- You are asked to design an experiment to confirm that T changes between a 10° and a 45° amplitude. You have a stopwatch which reads to the nearest 0.01 s. Assume that your reaction time is 0.1 s. You decide to time the pendulum over several oscillations and then divide the total time by the number of oscillations to arrive at T. How many oscillations would you need to measure to verify that T is longer at 45° than at 10°?



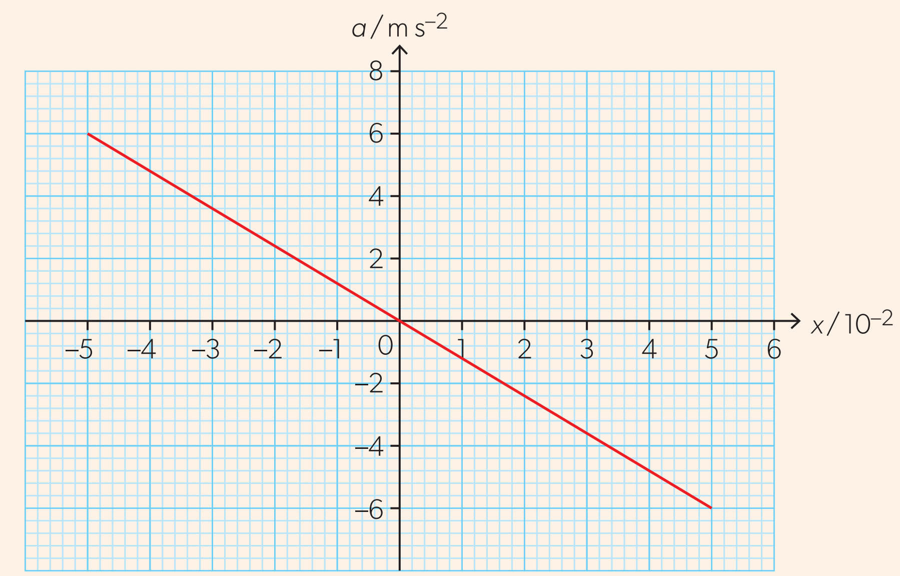
▲ Figure 9 A simple pendulum.

▲ Figure 10 Many mechanical objects can be approximated as either a mass on a spring or a pendulum. This picture shows a car's suspension which consists of a spring to absorb the shocks from bumps in the road. The car behaves like a mass on a spring and will have a time period for its oscillations.

Worked example 2

The graph shows how the acceleration a of an object varies with the displacement x.

a. Outline why the object performs simple harmonic motion.
b. State the amplitude of the oscillations.
c. Determine the period.



Solutions

a. The graph is a straight line with a negative slope through the origin. Hence, the acceleration is proportional to negative displacement and satisfies the defining equation of simple harmonic motion, $a = -\omega^2 x$.

b. The amplitude is equal to the maximum displacement, 5.0 cm.

c. The period is related to the angular frequency ω, which can be determined from the slope of the graph.

Slope $= -\omega^2 = -\frac{6.0}{5.0} \Rightarrow \omega = \sqrt{\frac{6.0}{5.0}} = 1.1\ \text{rad s}^{-1}$. From here, $T = \frac{2\pi}{\omega} = \frac{2\pi}{1.1} = 5.7\ \text{s}$.

Worked example 3

A mass of 0.045 kg oscillates simple harmonically at the end of a spring of spring constant 1.3 kN m^{-1}. Calculate the frequency of the oscillations.

Solution

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.045}{1.3 \times 10^3}} = 3.7 \times 10^{-2}\ \text{s}$.

$f = \frac{1}{T} = \frac{1}{3.7 \times 10^{-2}} = 27\ \text{Hz}$.

Worked example 4

An object of mass 2.1 kg attached to a spring undergoes simple harmonic motion on a horizontal frictionless surface. The period of oscillations is 1.8 s and the amplitude is 0.25 m.

Calculate:

a. the angular frequency

b. the maximum force acting on the object

c. the spring constant.

Solutions

a. $\omega = \frac{2\pi}{T} = \frac{2\pi}{1.8} = 3.5\ \text{rad s}^{-1}$.

b. From the defining equation of simple harmonic motion, the maximum acceleration of the object is $a_{max} = \omega^2 x_0$, where x_0 is the amplitude of oscillation. The maximum force is therefore $F_{max} = ma_{max} = m\omega^2 x_0 = (2.1)\left(\frac{2\pi}{1.8}\right)^2 (0.25) = 6.4\ \text{N}$.

c. $k = \frac{F_{max}}{x_0} = \frac{6.4}{0.25} = 26\ \text{N m}^{-1}$.

Practice questions

4. A force F acting on a point mass depends on the displacement x of the mass. Which of the relationships between F and x leads to simple harmonic motion?

A. $F = -x^2$ B. $F = -2x$ C. $F = 3x$ D. $F = 4x^2$

5. Calculate:

a. the period of a simple pendulum whose length is 0.80 m

b. length of a simple pendulum whose period is 2.4 s.

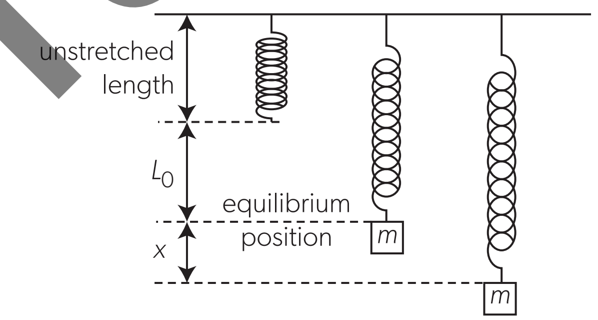
6. An object of mass 0.45 kg is attached to a spring with spring constant 12 N m^{-1}. The object undergoes simple harmonic motion with an amplitude of 0.15 m. Calculate:

a. the period of oscillation

b. the maximum force acting on the object from the spring.

7. A mass–spring system undergoes simple harmonic oscillations of a frequency 0.58 Hz. The mass is 0.90 kg. Calculate the spring constant.

8. A weightless spring of spring constant $k = 2.9\ \text{N m}^{-1}$ hangs vertically with a mass $m = 0.050$ kg attached to its free end. When the mass in in the equilibrium position, the spring extends by a distance L_0 relative to the unstretched length.



a. Calculate L_0.

The mass is displaced vertically from the equilibrium position by a distance x and released.

b. Draw a free-body diagram for the mass at the displaced position.

c. Show that the magnitude of the net force acting on the mass is kx.

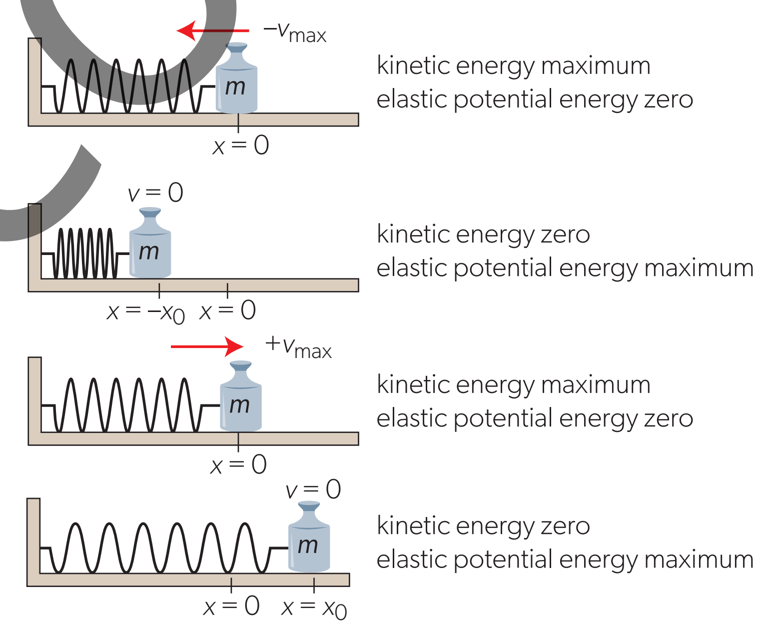
d. Compare the period of the vertical mass–spring system to that of a horizontal system, if the mass and the spring are the same in both systems.

e. Calculate the period of the oscillations.

Energy changes during simple harmonic motion

One way to interpret simple harmonic motion is in terms of energy transfer.

Figure 11 shows the transfers that occur in the horizontal mass–spring system.



▲ Figure 11 The energy transfers that occur in simple harmonic motion for a mass–spring system.

The mass is oscillating between $-x_0$ and $+x_0$. The amplitude of the motion is x_0. At each extreme, the speed of the mass is zero, so the kinetic energy is also zero. At this point, all the energy is in the form of stored elastic potential energy. At the centre of the motion the spring is at its natural (unextended) length and the mass is moving at its fastest, so the kinetic energy is also at a maximum with no energy stored in the form of elastic potential energy.

During one cycle of the oscillation, there are two kinetic-energy maxima because there are two velocity maxima, one in each direction when the mass is at the equilibrium position. In the same way, there are two maxima of elastic potential energy. The frequency of the energy transfers is double that of the frequency of the oscillation itself. Conversely, the time period for one energy cycle is half that of the time period for the simple harmonic motion.

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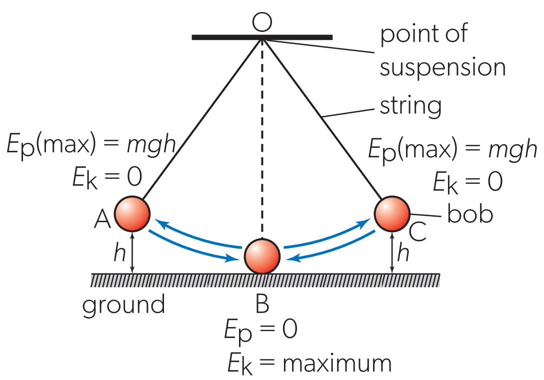
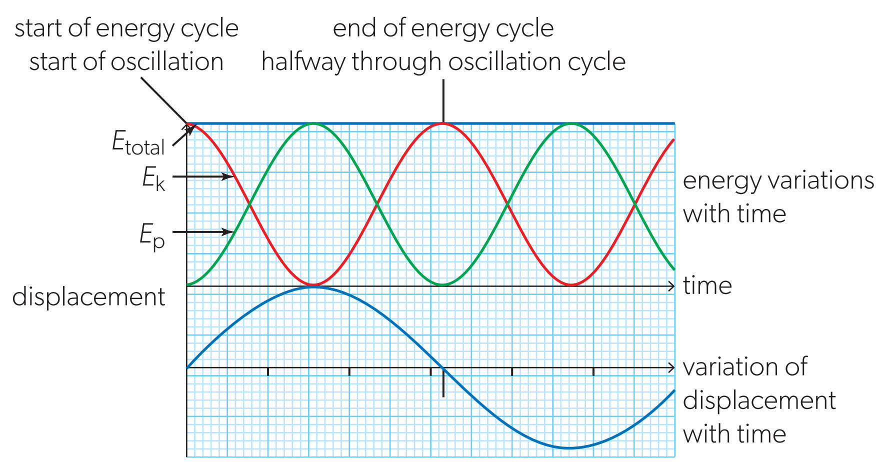


▲ Figure 12 The energy analysis is similar for the simple pendulum. The transfers between gravitational potential energy and kinetic energy for the pendulum bob are shown here.

Figure 12 shows the energy transfers for the simple pendulum.

For both oscillators, there is a continuous transfer between the kinetic and potential energies. When there are no energy losses from a system, such as those due to air resistance or friction, then the total energy in the system must be constant.

Figure 13 shows three graphs for the variation with time of the kinetic E_k, potential E_p and total energies E_{tot} for simple harmonic motion. It also shows how the displacement varies with time, so that the difference between the period of energy transfer and the period of simple harmonic motion is clear.



▲ Figure 13 The variations of kinetic and potential energies in simple harmonic motion with time. The total energy in the system is constant.

Worked example 5

A body undergoes simple harmonic motion of a frequency 20 Hz. How many times during one second is the kinetic energy of the body zero?

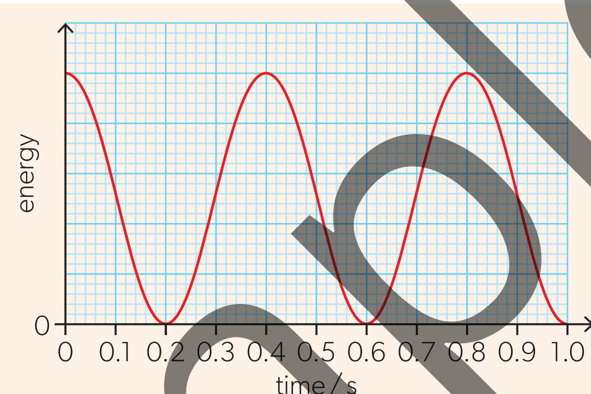
Solution

The KE is zero twice during one oscillation; hence $2 \times 20 = 40$ times per second.

Worked example 6

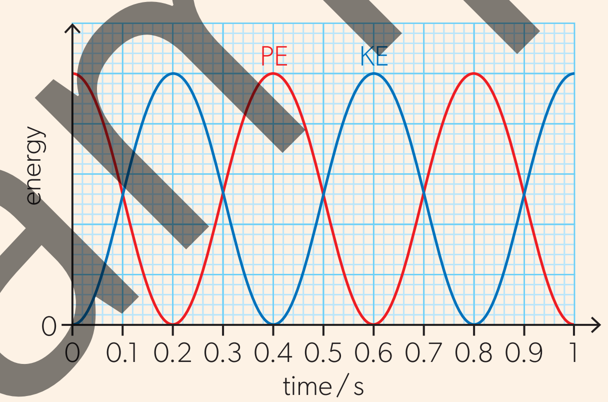
The graph shows how the potential energy of a simple pendulum varies with time.

a. Identify the first time when:
   i. the pendulum passes through the equilibrium position
   ii. the kinetic and the potential energies are equal.
b. State the period of oscillations.
c. Draw a graph of the variation of the kinetic energy of the pendulum with time.



Solutions

a. i. In the equilibrium position the potential energy is zero. This happens for the first time at 0.2 s.
   ii. The potential energy must decrease to one half of its maximum value. This happens at 0.1 s.
b. It takes 0.4 s to move from one extreme position (of maximum amplitude and potential energy) to the other. This is one half of the complete oscillation. The period is therefore $2 \times 0.4 = 0.8$ s.
c. The KE is a maximum when the PE is zero, and vice versa.



Energy loss and simple harmonic motion

Strictly speaking, once resistive losses of any sort occur for an oscillating system, then the oscillation is no longer simple harmonic. True simple harmonic motion never stops. The graphs for the variation with time of displacement/velocity/acceleration and the energy–time graphs have constant amplitudes as there are no resistance or energy losses to reduce the amplitude.

How can greenhouse gases be modelled as simple harmonic oscillators?
What physical explanation leads to the enhanced greenhouse effect? (NOS)

Topic B.2 gives the absorption mechanisms of electromagnetic radiation by molecules of the greenhouse gases. These molecules have vibrational states that are excited by the radiation. This leads to the temporary storage of the electromagnetic energy with subsequent re-radiation in different directions. There are links here both to the work in this topic but also to the resonance effects discussed in more detail in Topic C.4.

When changes to the atmosphere occur, then the levels of radiation absorption reflect the change. With greater concentrations of greenhouse gases, the absorption and re-radiation increases, leading to climate change.

How does the creation of links within physics enable scientists to develop greater understanding of the linked topics?

How can the understanding of simple harmonic motion apply to the wave model? (NOS)

There are strong links from Topic C.1 to the physics of Topics C.2 and C.3. Wave motion is a common phenomenon and a working knowledge of the mathematics of simple harmonic motion helps our understanding of wave behaviour and vice versa.

One way to describe the motion of a particle in a wave is in terms of a vector of constant length that rotates at a constant speed. Such a vector is known as a "phasor". This is the function of the red arrow in Figure 14. The arrowhead of the phasor traces out the motion of the wave particle. Wave motion and simple harmonic motion are closely interlinked, with the same terms and quantities being used in both.

Do links such as these give us further insights into the physical world?

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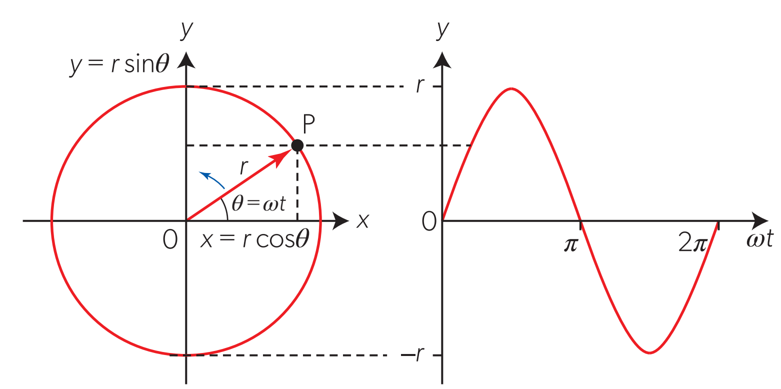
Linking circular motion and simple harmonic motion

When a circular motion in a horizontal plane is projected onto a vertical plane as in Figure 6, it is equivalent to a motion that is simple harmonic (Figure 14).



▲ Figure 14 Projecting circular motion onto a y-axis.

The y-axis point P is moving around the circle in Figure 14 at a constant angular speed ω.

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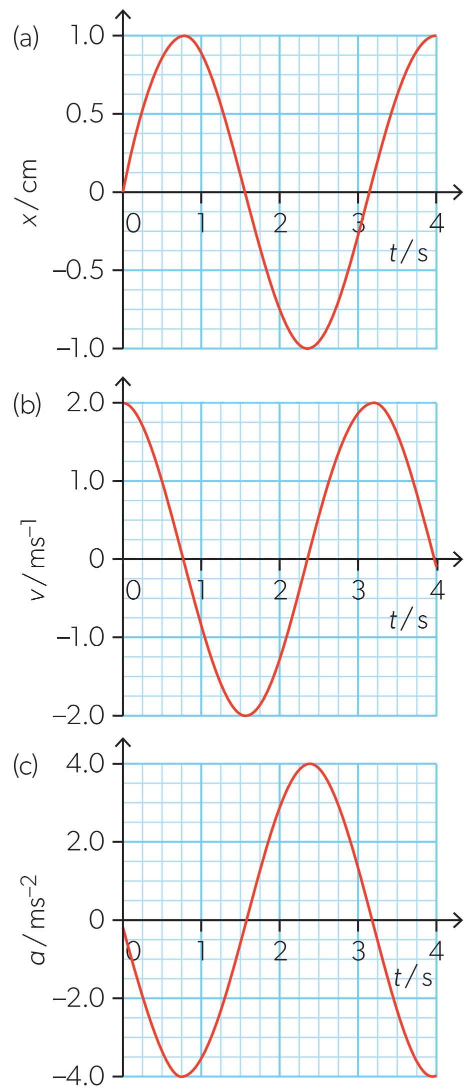

▲ Figure 15 Variation with time of (a) displacement, (b) velocity and (c) acceleration. These graphs all assume that the motion starts at the equilibrium position.

Two equations relate x and y to the circle of radius r and the angle θ between P and the x-axis. These are:

- $x = r\cos\theta$
- $y = r\sin\theta$.

The angle θ between the arrow and the x-axis is known as the **phase angle**.

As $\theta = \omega t$, the equations for the projection of P onto the diameter of the circle along the x-axis become $x = r\cos\omega t$ or $y = r\sin\omega t$. The radius of the circle is the amplitude of the simple harmonic motion so $r = x_0$ and we obtain the simple harmonic motion equations:

- $x = x_0\cos\omega t$ for simple harmonic motion that begins at the extremes
- $x = x_0\sin\omega t$ for simple harmonic motion that begins in the centre.

Two further equations also follow from the definition of simple harmonic motion and from $x = x_0\sin\omega t$:

The velocity $v = \frac{dx}{dt} = \omega x_0\cos\omega t$ and the acceleration $a = \frac{dv}{dt} = -\omega^2 x_0\sin\omega t$.

Notice that, because $x = x_0\sin\omega t$, then $a = -\omega^2(x_0\sin\omega t) = -\omega^2 x$. Our solution for the simple harmonic motion equation that arises from the projected circular motion satisfies the defining equation.

The three equations lead to three graphs.

Figure 15 shows the variations with time of (a) displacement, (b) velocity and (c) acceleration for the case where the motion starts at the centre. The displacement graph (a) is a sine curve, (b) is a cosine curve and (c) is a negative sine curve. (For motion starting at the positive extreme, they will be respectively (a) cosine, (b) −sine and (c) −cosine.)

The gradient at a particular time for the velocity–time graph gives the acceleration at that instant, and, similarly, the gradient of the displacement–time graph yields the velocity at that moment. This is easy to see at the extremes when the motion is momentarily at rest ($v = 0$).

There is another equation for the velocity that you will find useful because it does not contain t. Using the identity, $\sin^2\theta + \cos^2\theta = 1$, so that $\cos\theta = \pm\sqrt{1-\sin^2\theta}$ and substituting this into the speed equation gives $v = \pm\omega x_0\sqrt{1-\sin^2\theta}$. However, $\sin\theta = \frac{x}{x_0}$. To see why, look at Figure 15 and notice that $\sin\theta$ is the ratio of the displacement of P (which is at x) to the radius of the circle (which corresponds to the amplitude x_0). This gives $v = \pm\omega x_0\sqrt{1-\frac{x^2}{x_0^2}}$, which re-arranges to

$$v = \pm\omega\sqrt{x_0^2 - x^2}$$

The ± sign reminds us that the object can be travelling in either direction at a particular x. As you can see, this is a useful equation when you know the amplitude and displacement of an object but do not know the time at which the displacement occurs.

Displacement	$x = x_0\sin\omega t$
Velocity (x unknown)	$v = \omega x_0\cos\omega t$
Velocity (t unknown)	$v = \pm\omega\sqrt{x_0^2 - x^2}$
Acceleration	$a = -\omega^2(x_0\sin\omega t) = -\omega^2 x$

▲ Table 1 The four equations for simple harmonic motion.

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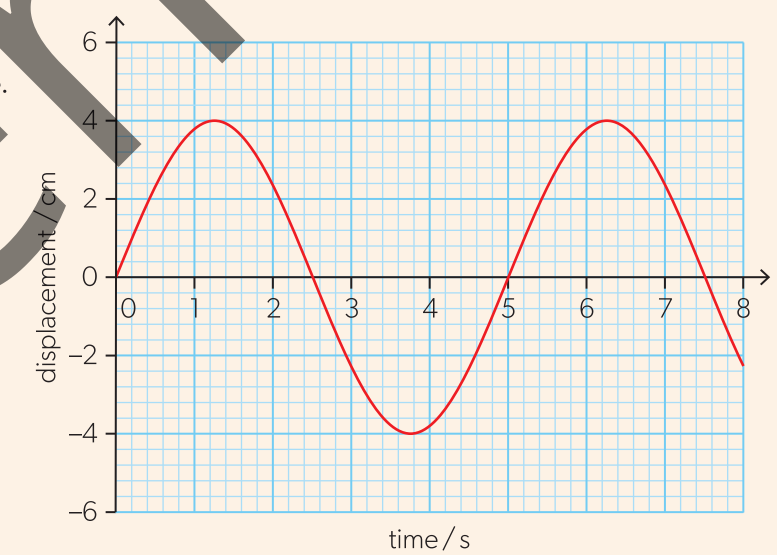
Worked example 7

The graph shows how the displacement of a body performing simple harmonic motion varies with time.



Calculate:

a. the angular frequency of oscillations
b. the maximum velocity of the body
c. the velocity after 3.0 s
d. the maximum acceleration.

Solutions

a. The period is 5.0 s. $\omega = \frac{2\pi}{5.0} = 1.3\,\text{rad s}^{-1}$.

b. The amplitude is 4.0 cm.
$v_{max} = \omega x_0 = \frac{2\pi}{5.0} \times 4.0 = 5.0\,\text{cm s}^{-1}$.

c. The displacement follows a sine function, $x = x_0\sin\omega t$. Hence, the velocity after a time t should be modelled with a cosine function, $v = \omega x_0\cos\omega t$.
At $t = 3.0\,\text{s}$, $v = \frac{2\pi}{5.0} \times 4.0\cos\left(\frac{2\pi}{5.0} \times 3.0\right) = -4.1\,\text{cm s}^{-1}$.

d. $a_{max} = \omega^2 x_0 = \left(\frac{2\pi}{5.0}\right)^2 \times 4.0 = 6.3\,\text{cm s}^{-2}$.

Worked example 8

A particle of mass 4.0 g undergoes simple harmonic motion with frequency 25 Hz and amplitude 13 mm.

Calculate, when the displacement of the particle is 10 mm:

a. the speed
b. the force acting on the particle.

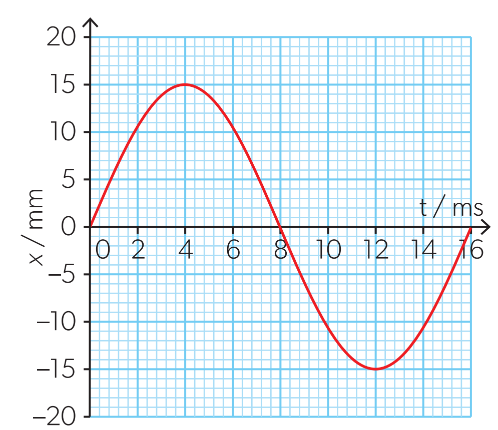
Solutions

a. The angular frequency is $\omega = \frac{2\pi}{T} = 2\pi f = 2\pi \times 25 = 157\,\text{rad s}^{-1}$.
$v = \omega\sqrt{x_0^2 - x^2} = 157\sqrt{13^2 - 10^2} = 1300\,\text{mm s}^{-1} = 1.3\,\text{m s}^{-1}$.

b. $F = ma = -m\omega^2 x = -4.0 \times 10^{-3} \times 157^2 \times 10 \times 10^{-3} = -0.99\,\text{N}$.

Practice questions

9. The graph shows how the displacement x of a particle undergoing simple harmonic motion varies with time t.



a. Identify the time when the particle has:
   i. the maximum negative velocity
   ii. the maximum positive acceleration.
b. Calculate the velocity of the particle:
   i. at $t = 10\,\text{ms}$
   ii. when $x = 5.0\,\text{mm}$ for the first time.
c. Calculate the maximum acceleration of the particle.

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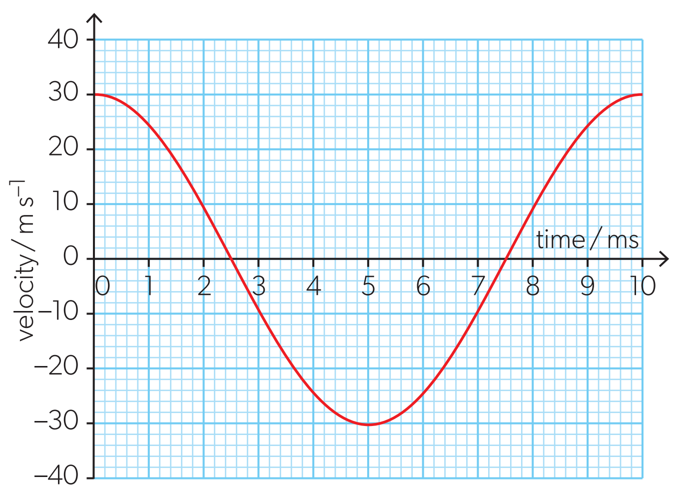
10. The velocity–time graph for an object undergoing simple harmonic motion is shown.



a. Identify the time at which the object has a maximum positive displacement.
b. Calculate the amplitude.
c. Calculate the displacement of the object at 4.0 ms.

11. An object of mass 100 g is suspended from a vertical spring of spring constant 7.8 N m^{-1}. The object is displaced by 12 cm vertically downwards from the equilibrium position and released.
a. Calculate the frequency of the oscillations.
b. Calculate the maximum speed of the object.
c. Calculate the speed of the object when it is 6.0 cm above the equilibrium position.

Modelling simple harmonic motion

- Tool 2: Use computer modelling.

The defining equation for simple harmonic motion is

$$a = -\omega^2 x$$

This can be written in differential form as $\frac{d^2x}{dt^2} = -\omega^2 x$ because acceleration is $\frac{d^2x}{dt^2}$. This second-order differential equation can be solved by calculus, by spreadsheet modelling or by using modelling software. This is one of many examples in physics of a simple second-order differential equation of the sort that you may meet in IB Diploma programme mathematics.

Simple harmonic motion is used as an example of modelling using a spreadsheet or modelling software in the section on Tools for physics (p. XXX).

Phase angle and phase difference

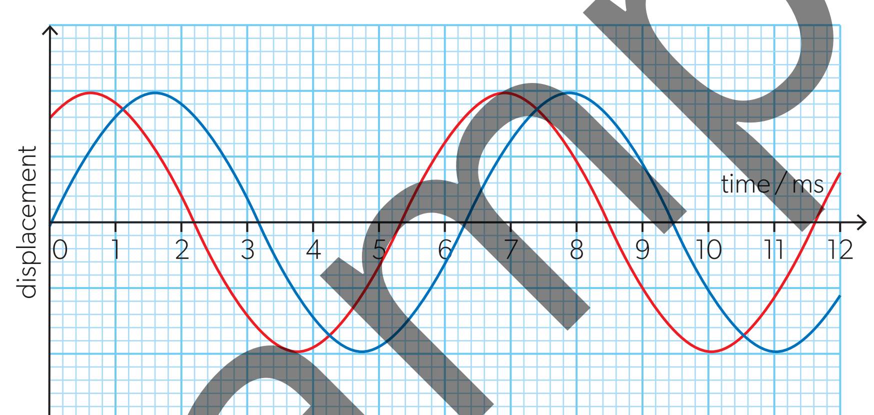
So far, we have looked at simple harmonic motion that begins at particular positions in the motion, the extreme displacements when $x = x_0$ and at the centre of the motion when $x = 0$. Is it possible to produce an equation that allows for any starting point?

The simple harmonic motion equation is a second-order differential equation, and it can be shown that there are general solutions to this equation. One of these is

$$x = x_0 \sin(\omega t + \phi)$$

This resembles the earlier solutions, but has the addition of the single term ϕ. This quantity is known as the **phase angle** as before.

Look carefully at the displacement–time graphs for two simple harmonic motions in Figure 16. At the beginning of the graph, the blue curve shows a displacement of zero, but the red curve is just about to reach its maximum displacement. It is about one-eighth of a cycle ahead of the displacement. To be precise, the red curve is one radian ahead of the blue curve. As one cycle corresponds to 2π rad, the red curve leads the blue by $\frac{1}{2\pi} = \frac{1}{6.3} = 0.16$ of a cycle.

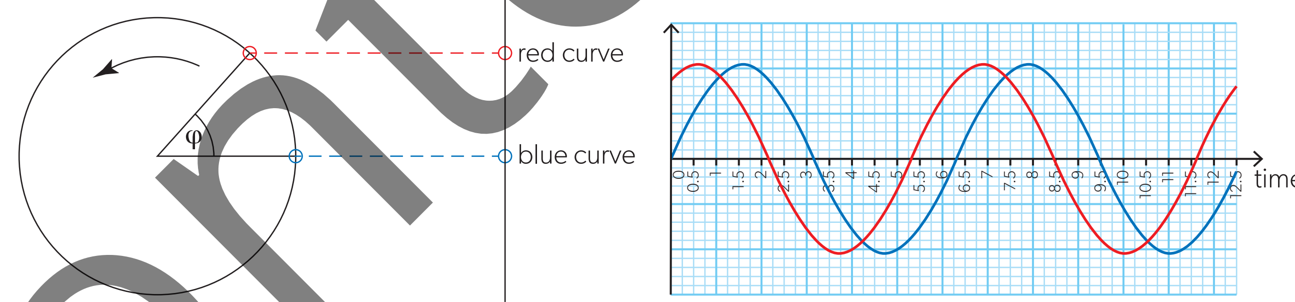


▲ Figure 16 Phase difference in simple harmonic motion. The blue curve lags behind the red curve because it reaches its peak at a later time.

AHL

Since the equation for the blue curve is $x = x_0 \sin(\omega t + 0)$, then the equation for the red curve must be $x = x_0 \sin(\omega t + 1.0)$.

The phase difference between the curves in Figure 16 can be modelled using the circular motions for both oscillations, as in Figure 17. Remember that both oscillations have the same ω and therefore travel around the circle at the same angular speed. The **phase difference** is the angle between the radial lines that are tracing out the simple harmonic motion as the blue tracing point chases the red point around the circle.



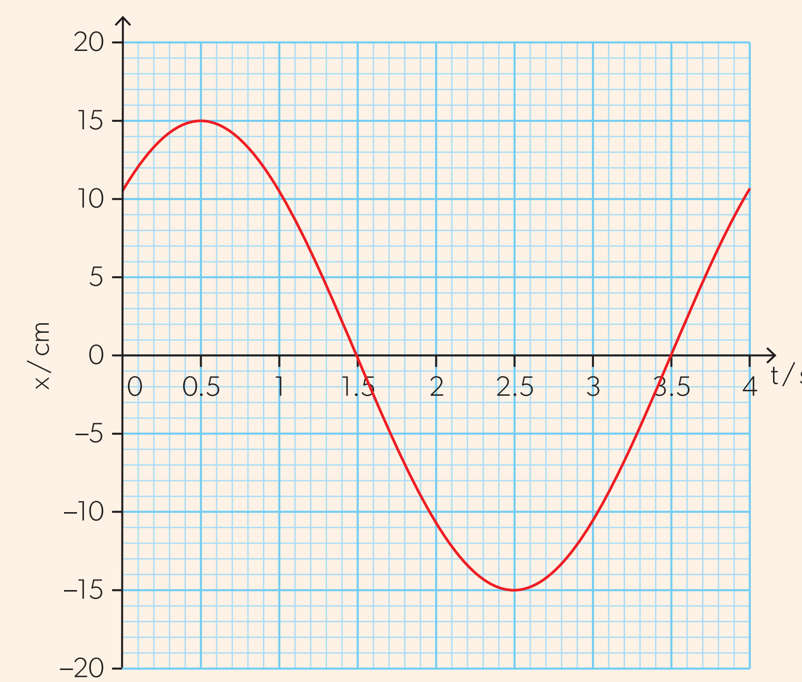
▲ Figure 17 A circular motion projected onto a line gives simple harmonic motion. The red point leads the blue point by one radian.

The equations for displacement, velocity and acceleration in full become:

- Displacement: $x = x_0 \sin(\omega t + \phi)$
- Velocity: $v = \omega x_0 \cos(\omega t + \phi)$
- Acceleration: $a = -\omega^2 x = -\omega^2 x_0 \sin(\omega t + \phi)$

Worked example 9

The graph shows how the displacement x varies with time t for an object undergoing simple harmonic motion.



a. The displacement can be modelled with an equation $x = x_0 \sin(\omega t + \phi)$.
   i. State the value of x_0.
   ii. Calculate the value of ω.
   iii. Determine the phase angle ϕ.
b. Calculate the velocity of the object at $t = 3.0$ s.

Solutions

a. i. $x_0 = 0.15$ m.
   ii. The period of motion is 4.0 s.
   $\omega = \frac{2\pi}{4.0} = 1.6$ rad s^{-1}.
   iii. The object is at the equilibrium position after 1.5 s. Had the oscillation started at $x = 0$, the object would have returned to the equilibrium position after 2.0 s, which is $\frac{1}{8}$ of the period later than it actually did. The phase angle is therefore $\phi = \frac{2\pi}{8} = \frac{\pi}{4} \simeq 0.79$ rad.

b. $v = \omega x_0 \cos(\omega t + \phi) = \left(\frac{2\pi}{4.0}\right)(0.15)\cos\left(\frac{2\pi}{4.0} \times 3.0 + \frac{\pi}{4}\right) = 0.17$ m s^{-1}.

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Worked example 10

The displacement x, in metres, of a particle undergoing simple harmonic motion is given by the equation $x = 7.5 \times 10^{-3}\sin(12t + 2.0)$, where t is the time in seconds.

a. Calculate the period of motion.
b. Calculate the velocity of the particle after 0.30 s.

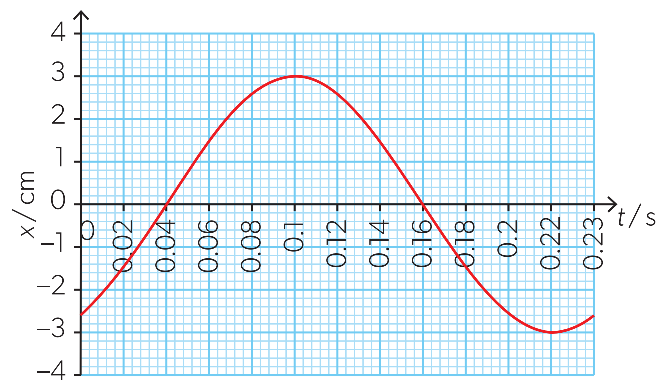
Solutions

a. The angular frequency is $\omega = 12\ \text{rad s}^{-1}$. $T = \frac{2\pi}{\omega} = \frac{2\pi}{12} = 0.52\ \text{s}$.

b. $v = \omega x_0 \cos(\omega t + \phi) = 12 \times 7.5 \times 10^{-3}\cos(12 \times 0.3 + 2.0) = 7.0 \times 10^{-2}\ \text{m s}^{-1}$.

Practice questions

12. The graph shows the variation with time t of the displacement x of a particle undergoing simple harmonic motion.

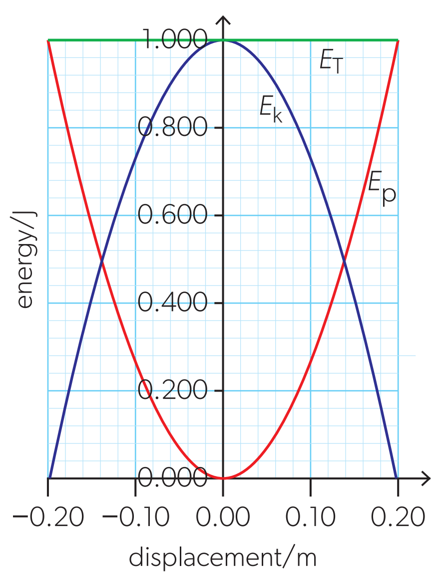


The oscillation can be modelled with an equation $x = x_0\sin(\omega t + \phi)$.

a. Determine the values of x_0, ω and ϕ.
b. Calculate the maximum velocity of the particle.
c. Calculate the velocity and the acceleration of the particle after 0.08 s.

13. The displacement x, in cm, of a particle undergoing simple harmonic motion is given by $x = 12.0\sin(0.500t + 1.00)$, where t is time in s.

a. Calculate the period of the oscillations.
b. Calculate the velocity at $t = 0$.



▲ Figure 18 E_k, E_p and E_T for simple harmonic motion.

Energy transfer equations

The energy transfers between kinetic E_k and potential E_p that drive harmonic oscillators were described earlier. The simple harmonic motion equations can be used to derive a set of energy equations. Phase differences are ignored, but ϕ can easily be re-introduced into the equations when you need to.

The kinetic equation is related to $\frac{1}{2}mv^2$ as usual. The speed $v = \pm\omega\sqrt{x_0^2 - x^2}$ and therefore $v^2 = \omega^2(x_0^2 - x^2)$, so that

$$E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2)$$

where m is the mass of the object undergoing simple harmonic motion.

Immediately, we can see that the total energy (which occurs when the object is moving at its fastest when $x = 0$) is

$$E_{tot} = \frac{1}{2}m\omega^2 x_0^2$$

Also, $E_{tot} = E_k + E_p$ and therefore $E_p = E_{tot} - E_k = \frac{1}{2}m\omega^2 x_0^2 - \frac{1}{2}m\omega^2(x_0^2 - x^2)$

which is equal to $\frac{1}{2}m\omega^2 x_0^2 - \frac{1}{2}m\omega^2 x_0^2 + \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 x^2$.

AHL

The graphs of the variations of both E_k and E_p with displacement are parabolas. Figure 18 shows E_{tot}, E_k and E_p all plotted against displacement.

Notice that the displacement at which the kinetic energy and the potential energy are equal ($E_k = E_p$) is not at half the amplitude but closer to x_0 than the equilibrium point.

Worked example 11

A mass of 0.15 kg attached at the end of a weightless spring oscillates with simple harmonic motion. The mass passes through the equilibrium position with a speed of 1.4 m s⁻¹.

a. Calculate the total energy of the oscillating system.
b. The spring constant is 6.4 N m⁻¹. Determine the amplitude of the oscillations.

Solutions

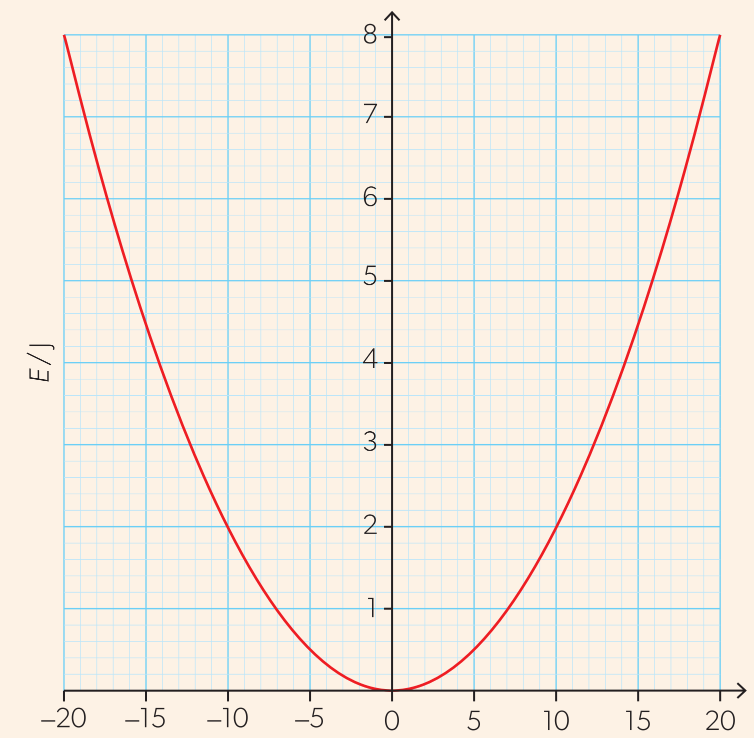
a. At the equilibrium position, the potential energy is zero, so the total energy of the system is kinetic only. $E_T = \frac{1}{2}mv^2 = \frac{1}{2}(0.15)(1.4)^2 = 0.147\ \text{J}$.

b. We can find the amplitude x_0 by re-arranging the equation $E_T = \frac{1}{2}m\omega^2 x_0^2 \Rightarrow x_0 = \sqrt{\frac{2E_T}{m\omega^2}}$. For a mass–spring system, we have $a = -\frac{k}{m}x$ and so $\omega^2 = \frac{k}{m}$. We combine the equations to get $x_0 = \sqrt{\frac{2E_T}{k}} = \sqrt{\frac{2 \times 0.147}{6.4}} = 0.21\ \text{m}$.

Worked example 12

The graph shows how the potential energy of an object executing simple harmonic motion varies with the displacement of the object. The amplitude of motion is 20 cm.

a. State the total energy of the oscillating system.
b. Estimate, using the graph, the displacement of the object when the kinetic and the potential energies are equal.
c. Sketch a graph showing the variation of the kinetic energy of the object with displacement.
d. The mass of the object is 2.6 kg. Calculate the maximum speed of the object.
e. Determine the period of the oscillations.



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Solutions

a. 8.0 J

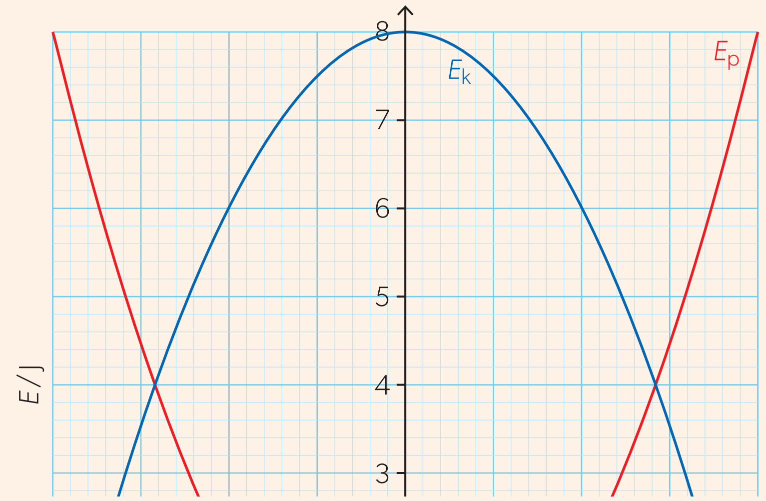
b. In this situation both E_k and E_p are equal to 4.0 J. From the graph, this happens when the displacement is approximately ±14 cm.

c. The graph has a similar parabolic shape but is inverted compared with the potential energy curve.

d. The maximum kinetic energy is 8.0 J, so the maximum speed can be calculated from

$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 8.0}{2.6}} = 2.5\ \mathrm{m\,s^{-1}}$$

e. It is convenient to first find the angular frequency and then the period T.

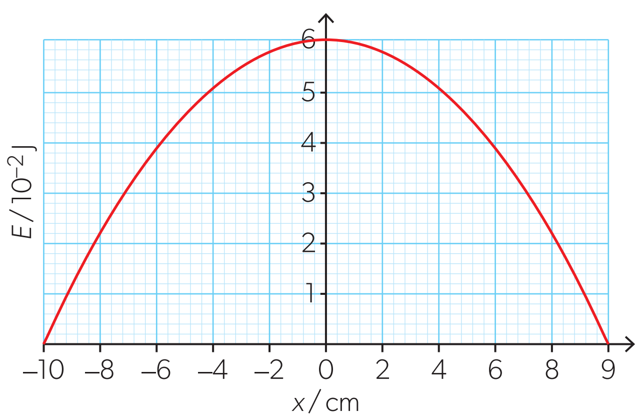


$$E_T = \frac{1}{2} m\omega^2 x_0^2 \Rightarrow \omega = \sqrt{\frac{2E_T}{mx_0^2}} = \sqrt{\frac{2 \times 8.0}{2.6 \times 0.20^2}} = 12.4\ \mathrm{rad\,s^{-1}}. \text{ From here, } T = \frac{2\pi}{12.4} = 0.51\ \mathrm{s}.$$

Practice questions

14. An object of mass 0.060 kg undergoes simple harmonic motion with frequency 4.0 Hz and amplitude 0.25 m. Calculate, when the displacement of the object is 0.10 m:
    a. the potential energy
    b. the kinetic energy.

15. The graph shows how the kinetic energy of an oscillating mass-spring system varies with the displacement of the mass from the equilibrium position. The mass is 0.70 kg.



    a. Calculate the maximum velocity of the mass.
    b. Determine:
        i. the period of the oscillations
        ii. the spring constant.

16. An object of mass 1.00 kg is attached to a spring with spring constant $4.50 \times 10^2\ \mathrm{N\,m^{-1}}$ and is allowed to undergo simple harmonic motion on a frictionless horizontal surface. The object is initially displaced by 0.200 m and is given an initial velocity of 3.50 m s⁻¹. Determine:
    a. the total energy of the system
    b. the amplitude of the oscillations.

17. An object oscillates simple harmonically with an amplitude x_0. When the displacement of the object is zero, the kinetic energy of the object is E. What is the kinetic energy of the object when the displacement is $\frac{x_0}{2}$?

    A. $\frac{E}{4}$ B. $\frac{E}{2}$ C. $\frac{3E}{4}$ D. E

It is useful to repeat the comments about energy variation with time from page 329 algebraically:

A substitution from the simple harmonic motion velocity equation gives $E_k = \frac{1}{2} m(\omega x_0 \cos\omega t)^2$ which becomes $E_k = \frac{1}{2} m\omega^2 x_0^2 \cos^2\omega t$.

Using $E_p = E_{tot} - E_k$ leads to $E_p = \frac{1}{2} m\omega^2 x_0^2 - \frac{1}{2} m\omega^2 x_0^2 \cos^2\omega t$. This simplifies to $\frac{1}{2} m\omega^2 x_0^2 (1 - \cos^2\omega t)$ and hence, using $\sin^2\theta + \cos^2\theta = 1$, to

$$E_p = \frac{1}{2} m\omega^2 x_0^2 \sin^2\omega t.$$

The energy–time graphs in Figure 13 showed the relationships between E_{tot}, E_p and E_k, and remind you that the frequency of the energy change is double the frequency of the underlying simple harmonic motion.

Total energy E_{tot}	$\frac{1}{2} m\omega^2 x_0^2$	$\frac{1}{2} m\omega^2 x_0^2$
Potential energy E_p	$\frac{1}{2} m\omega^2 x^2$	$\frac{1}{2} m\omega^2 x_0^2 \sin^2\omega t$
Kinetic energy E_k	$\frac{1}{2} m\omega^2 (x_0^2 - x^2)$	$\frac{1}{2} m\omega^2 x_0^2 \cos^2\omega t$

▲ Table 2 The energy equations.

How does damping affect periodic motion?

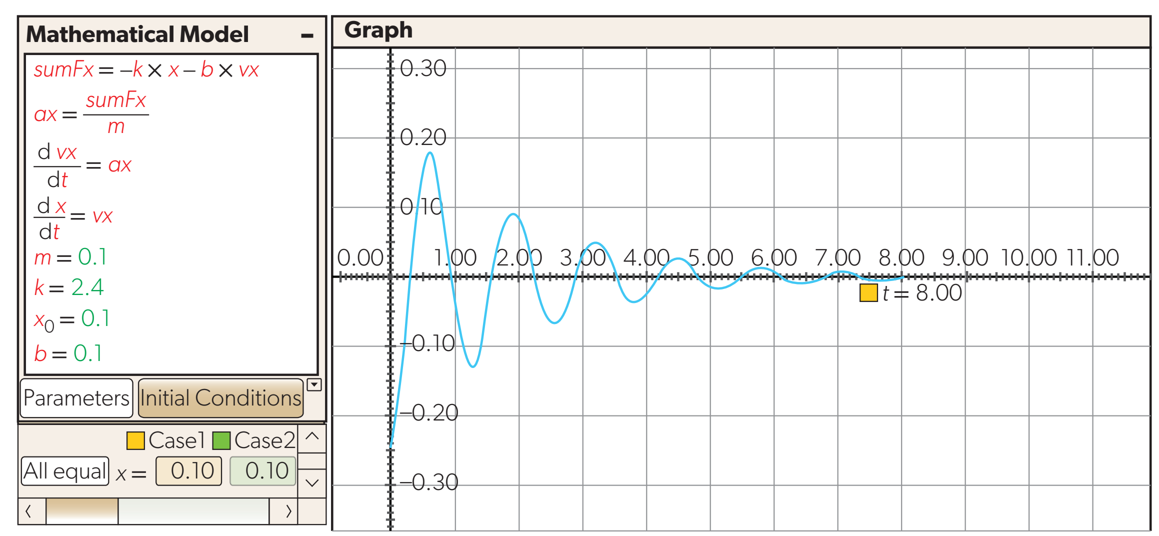
Strictly speaking, once resistive losses of any sort occur for an oscillating system, then the oscillation is no longer simple harmonic. The graphs for the variation with time of displacement/velocity/acceleration and the energy–time graphs have constant amplitudes, as there are no resistance or energy losses to reduce the amplitude. This is discussed in more detail in Topic C.4,where the effects of damping (friction) are described in detail.

This is an easy question to answer if you use modelling software, as shown in the section on modelling in *Tools for physics* p XXX. In the Modellus X model used there, only one change is required to the first equation.

The term $-b \times vx$ must be added to represent a drag force that is proportional to the speed. The drag coefficient is *b*; *vx* is the velocity of the oscillating particle.

When *b* is set to 1.0 ,then the behaviour of the oscillating system changes to an oscillation that is damped. The amplitude decreases with time, and the motion eventually stops.

You can explore the effects of varying *d* if you set this model up for yourself. A particularly interesting case occurs with $b = 2.6$. This is **critical damping** and is examined in Topic C.4.



◀ Figure 19 Part of the ModellusX software screen running a model of damped simple harmonic motion and the outcome when the model is run. The graph is displacement against time.

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